S2 S13 UK

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1. A bag contains a large number of 1p, 2p and 5p coins.

50% are 1p coins
20% are 2p coins
30% are 5p coins

A random sample of 3 coins is chosen from the bag.

(a) List all the possible samples of size 3 with median 5p. **(2)**

(b) Find the probability that the median value of the sample is 5p. **(4)**

(c) Find the sampling distribution of the median of samples of size 3 **(5)**

a) 5,5,5
5,5,1   5,1,5   1,5,5
5,5,2   5,2,5   2,5,5

b) $P(5,5,5) = 0.3^3$
$P(2\,5s,1) = 3 \times 0.3^2 \times 0.5$
$P(2\,5s,2) = 3 \times 0.3^2 \times 0.2$
$P(Q_2 = 5) = \frac{27}{125}$

c) $P(Q_2 = 1) = 0.5^3 + 3 \times 0.5^2 \times 0.2 + 3 \times 0.5^2 \times 0.3 = 0.5$

$P(Q_2 = 2) = 0.2^3 + 3 \times 0.2^2 \times 0.5 + 3 \times 0.2^2 \times 0.3 + 0.5 \times 0.2 \times 0.3 \times 6$

$= \frac{71}{250}$

1,2,5
1,5,2
2,5,1
2,1,5
5,2,1
5,1,2

Q_2	1	2	5
P	0.5	0.284	0.216

2. The number of defects per metre in a roll of cloth has a Poisson distribution with mean 0.25

Find the probability that

(a) a randomly chosen metre of cloth has 1 defect,

(2)

(b) the total number of defects in a randomly chosen 6 metre length of cloth is more than 2

(3)

A tailor buys 300 metres of cloth.

(c) Using a suitable approximation find the probability that the tailor's cloth will contain less than 90 defects.

(5)

a) X = # defects in 1m roll $\quad X \sim Po(0.25)$

$P(X=1) = \dfrac{e^{-0.25} \times 0.25^1}{1} = 0.195 \ (3sf)$

b) y = # defects in 6m roll $\quad y \sim Po(1.5)$

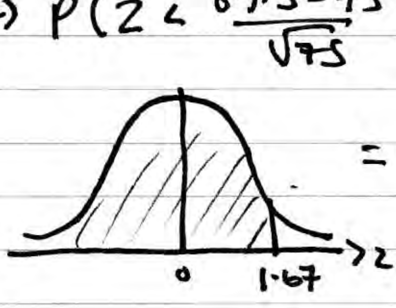
$P(y>2) = P(y \geq 3) = 1 - P(y \leq 2) = 1 - 0.8088 = 0.1912$

c) $\mu = 300 \times 0.25 = 75 \quad \therefore \sigma^2 = 75$

t = defects in 300m $\quad P(t<90) \Rightarrow P(t \leq 89)$

$t \sim Po(75) \simeq t \sim N(75, 75)$ cc $P(t < 89.5)$

$\Rightarrow P\left(Z < \dfrac{89.5 - 75}{\sqrt{75}}\right) \approx P(Z < 1.67)$



$= \Phi(1.67) = 0.953$

3. An online shop sells a computer game at an average rate of 1 per day.

(a) Find the probability that the shop sells more than 10 games in a 7 day period.

(3)

Once every 7 days the shop has games delivered before it opens.

(b) Find the least number of games the shop should have in stock immediately after a delivery so that the probability of running out of the game before the next delivery is less than 0.05

(3)

In an attempt to increase sales of the computer game, the price is reduced for six months. A random sample of 28 days is taken from these six months. In the sample of 28 days, 36 computer games are sold.

(c) Using a suitable approximation and a 5% level of significance, test whether or not the average rate of sales per day has increased during these six months. State your hypotheses clearly.

(7)

a) x = games sold per 7 days $\quad x \sim Po(7)$

$P(x > 10) \Rightarrow P(x \geq 11) = 1 - P(x \leq 10) = 1 - 0.9015 = \underline{0.0985}$

b) $P(x > L) < 0.05$ $\quad P(x < 11) = 0.9467$

$1 - P(x \leq L) < 0.05$ $\quad P(x < 12) = 0.9730$ ✓

$P(x \leq L) > 0.95$ $\quad \therefore \underline{L = 12}$

c) y = games sold per 28 days $\quad y \sim Po(28)$

$\mu = 28, \sigma^2 = 28$

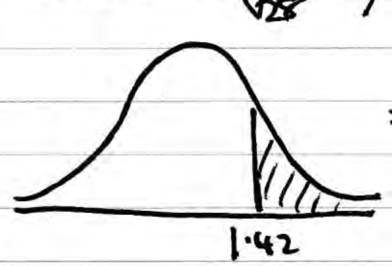
null hyp $H_0 : \lambda = 28$

alt hyp $H_1 : \lambda > 28$

$y \sim Po(28) \simeq y \sim N(28, 28)$ $\quad P(y \geq 36) \Rightarrow P(y > 35)$

cc $P(y > 35.5)$

$\simeq P\left(Z > \frac{35.5 - 28}{\sqrt{28}}\right) \simeq P(Z > 1.42)$



$= 1 - \Phi(1.42) = 0.0778 \ (> 0.05)$

∴ Not statistically significant

∴ not enough evidence to reject null hypothesis

∴ no evidence to suggest sales have increased

4. A continuous random variable X is uniformly distributed over the interval $[b, 4b]$ where b is a constant.

(a) Write down $\mathrm{E}(X)$.

(1)

(b) Use integration to show that $\mathrm{Var}(X) = \dfrac{3b^2}{4}$.

(3)

(c) Find $\mathrm{Var}(3 - 2X)$.

(2)

Given that $b = 1$ find

(d) the cumulative distribution function of X, $\mathrm{F}(x)$, for all values of x,

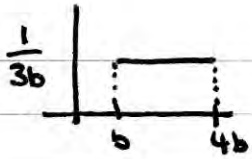
(2)

(e) the median of X.

(1)

a) $X \sim U[b, 4b]$ $\quad E(X) = \dfrac{4b+b}{2}$

$E(X) = \frac{5}{2}b$



b) $E(X^2) = \int x^2 f(x)\,dx = \int_b^{4b} \dfrac{x^2}{3b}\,dx = \left[\dfrac{x^3}{9b}\right]_b^{4b}$

$E(X^2) = \dfrac{64b^3 - b^3}{9b} = 7b^2$

$V(X) = E(X^2) - E(X)^2$

$V(X) = 7b^2 - \left(\frac{5}{2}b\right)^2 = \dfrac{3b^2}{4}$

c) $V(3-2X) = \cancel{3}\,2^2(V(X)) = 3b^2$

d) $b = 1 \Rightarrow f(x) = \frac{1}{3} \quad 1 \le x \le 4$

$F(x) = \int f(x)\,dx \Rightarrow \int_1^x \frac{1}{3}\,dt = \left[\frac{1}{3}t\right]_1^x = \frac{1}{3}x - \frac{1}{3}$

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{3}x - \frac{1}{3} & 1 \le x \le 4 \\ 1 & x > 4 \end{cases}$$

e) $F(Q_2) = 0.5 \Rightarrow \frac{1}{3}x - \frac{1}{3} = \frac{1}{2} \Rightarrow 2x - 2 = 3 \Rightarrow 2x = 5$

$x = 2.5$

5. The continuous random variable X has a cumulative distribution function

$$F(x) = \begin{cases} 0 & x < 1 \\ \dfrac{x^3}{10} + \dfrac{3x^2}{10} + ax + b & 1 \leqslant x \leqslant 2 \\ 1 & x > 2 \end{cases}$$

where a and b are constants.

(a) Find the value of a and the value of b. **(4)**

(b) Show that $f(x) = \dfrac{3}{10}(x^2 + 2x - 2), \quad 1 \leqslant x \leqslant 2$ **(1)**

(c) Use integration to find $E(X)$. **(4)**

(d) Show that the lower quartile of X lies between 1.425 and 1.435 **(3)**

a) $F(1) = 0 \Rightarrow \frac{1}{10} + \frac{3}{10} + a + b = 0 \quad \therefore a + b = -0.4$

$F(2) = 1 \Rightarrow \frac{8}{10} + \frac{12}{10} + 2a + b = 1 \quad \therefore 2a + b = -1$ −

$a = -0.6$

$b = 0.2$

b) $f(x) = \frac{d}{dx}F(x) = \frac{3}{10}x^2 + \frac{6}{10}x - \frac{6}{10}$

$= \frac{3}{10}(x^2 + 2x - 2) \quad 1 \leq x \leq 2$

c) $E(X) = \int x f(x)\,dx = \frac{3}{10}\int_1^2 x^3 + 2x^2 - 2x\,dx$

$= \frac{3}{10}\left[\frac{1}{4}x^4 + \frac{2}{3}x^3 - x^2\right]_1^2 = \frac{3}{10}\left[\frac{16}{3} - \left(-\frac{1}{12}\right)\right] = \frac{13}{8}$

d) $F(Q_1) = 0.25 \qquad F(1.425) = 0.244 < 0.25$

$F(1.435) = 0.253 > 0.25$

$\therefore \quad 1.425 < Q_1 < 1.435$

6. In a manufacturing process 25% of articles are thought to be defective. Articles are produced in batches of 20

(a) A batch is selected at random. Using a 5% significance level, find the critical region for a two tailed test that the probability of an article chosen at random being defective is 0.25

You should state the probability in each tail which should be as close as possible to 0.025

(5)

The manufacturer changes the production process to try to reduce the number of defective articles. She then chooses a batch at random and discovers there are 3 defective articles.

(b) Test at the 5% level of significance whether or not there is evidence that the changes to the process have reduced the percentage of defective articles. State your hypotheses clearly.

(5)

a) X = defective article in batch of 20

$X \sim B(20, 0.25)$

$P(x \leq L) \simeq 0.025$

$P(x \leq 1) = 0.0243$

$\therefore L = 1$

$P(X \geqslant U) \simeq 0.025$ $P(X > U-1)$

$1 - P(X \leq U-1) \simeq 0.025$

$\Rightarrow P(X \leq U-1) \simeq 0.975$

$P(X \leq 9) \simeq 0.9861$

$\therefore U - 1 = 9 \quad \therefore U = 10$

$CR \{x \leq 1\} \cup \{x \geqslant 10\}$

b) null hyp $H_0: P = 0.25$

alt hyp $H_1: P < 0.25$

$P(x \leq 3) = 0.2252 \ (>0.05)$

also 3 is not in CR.

$\therefore$ not enough evidence to reject null hypothesis as test was not statistically significant.

no evidence to suggest changes have reduced percentage of defective articles.

7. A telesales operator is selling a magazine. Each day he chooses a number of people to telephone. The probability that each person he telephones buys the magazine is 0.1

(a) Suggest a suitable distribution to model the number of people who buy the magazine from the telesales operator each day.

(1)

(b) On Monday, the telesales operator telephones 10 people. Find the probability that he sells at least 4 magazines.

(3)

(c) Calculate the least number of people he needs to telephone on Tuesday, so that the probability of selling at least 1 magazine, on that day, is greater than 0.95

(3)

A call centre also sells the magazine. The probability that a telephone call made by the call centre sells a magazine is 0.05
The call centre telephones 100 people every hour.

(d) Using a suitable approximation, find the probability that more than 10 people telephoned by the call centre buy a magazine in a randomly chosen hour.

(3)

a) Binomial, fixed number of trials n
constant probability $X \sim B(n, 0.1)$
each trial is independent.

b) $X \sim B(10, 0.1)$ $P(X \geq 4)$ $P(X > 3)$
$= 1 - P(X \leq 3) = 1 - 0.9872 = \underline{0.0128}$

c) $P(X \geq 1) > 0.95 \Rightarrow P(X = 0) < 0.05$
$\Rightarrow 0.9^n < 0.05$
$\Rightarrow n \log 0.9 < \log 0.05 \Rightarrow n > \dfrac{\log 0.05}{\log 0.9} \Rightarrow n > 28.4$

$\therefore$ least number of calls $= 29$

d) $= 5$ p/h $y =$ Sales per hour $y \sim Po(5)$

$P(y > 10)$ $P(y \geq 11)$ $= 1 - P(y \leq 10)$
$= 1 - 0.9863$
$= \underline{0.0137}$